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THE INITIAL GROWTH OF A SPHERICAL EXPLOSION IN SEA WATER

by

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Abstract

This report, which is a sequel to papers by Holt (1955 and 1956), describes the first stages of the calculation of the growth of a spherical explosion in sea water, due to a charge of PETN initiated at its center. Working in the time distance plane, the analysis of Holt (1955 and 1956) is used to construct a "plus" characteristic traversing the whole field of disturbance near the origin of blast and the "minus" characteristic which is the outer boundary of the detonation region. These initial data are then used to start a calculation of the further growth of the blast field by the numerical method of characteristics. This is programmed on an IBM CPC computer. A procedure is established which is satisfactory in principle but unsuitable for repeated application on this particular computer owing to inadequate storage facilities. As a consequence only the first stages of the calculation are carried out now and the work will be completed later on an IBM 704 computer.

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I. <u>Introduction</u>.

This report is a continuation of work by Holt (1955 and 1956) on the initial behavior of a spherical explosion. A beginning is made with the numerical calculation of the growth of a spherical explosion in sea water, due to a charge of PETN initiated at its center. The basic data are taken from Holt (1956), where series expansions are developed to determine the field of disturbance near the origin of blast (the point O' in the time distance plane shown in Figure 1).

Three stages preliminary to this calculation are completed in the present report. Firstly, the series expansions are used to calculate initial values of physical variables on a "plus" characteristic traversing the whole disturbance field near O'. Secondly, the ordinary differential equations satisfied in the detonation region are integrated, using Jones' equation of state for PETN; the integration leads to the contstruction of the minus characteristic dividing the detonation region from the gas expansion region. Thirdly, the data on these two characteristic lines are used to initiate a calculation of the remaining field of disturbance by the numerical method of characteristics. This work is programmed on an IBM CPC computer attached to the Division. The program is satisfactory in principle but requires a larger storage capacity than that available on this machine. Accordingly only the first stage of the numerical calculation has been carried out so far and the program will be completed on an IBM 704 computer which has the required storage facilities.

The physical phenomena and the theoretical analysis of the problem are given below briefly. The description is derived from the fuller account given in Berry and Holt (1954).

When a spherical charge of orthodox explosive is initiated at its center it is observed that the main blast wave is followed by a second blast wave which is very weak at first but attains an appreciable strength later. The cause of this second wave is the slightly excessive expansion of the explosive gases behind the detonation front which, if unchecked, would make the gas pressure at its outer boundary too low in comparison with the adjacent fluid pressure behind the powerful main blast wave. The second blast wave, which always starts inside the explosive gas region, has the effect of increasing the gas pressure sufficiently to ensure continuity at the fluid-gas boundary.

Theoretical analyses of the existence of the second blast wave have been done by Whitham (1950), Wecken (1951), Berry and Holt (195[illegible]), Berry, Butler and Holt (1954) and Holt (1955) based on the investigations made independently by Taylor (1950) and Döring (Döring and Burkhardt (1946)). The early development of the disturbance from a typical spherical explosion is illustrated in Figure 1, which shows trajectories in the t,r plane, where t is the time measured from the instant of initiation and r is the radial distance from the center of the explosion. It begins with a detonation phase, during which a strong detonation wave O'D travels outwards from the center and reacts on the solid explosive to produce highly compressed gases.

This is the main blast wave, which is followed by a region of disturbed fluid denoted by c. At the same time the gases released by the explosion expand rapidly through a centered expansion wave AO'B, a region denoted by e. At the head of this expansion wave is another gas region of initially uniform flow BO'C denoted by g, this is adjacent to the compressed fluid region c.

In the stage following the detonation process characteristic curves play an important part. We define a "minus characteristic" to be a line in the t,r plane, the slope of which is everywhere equal to the difference between the local velocity of fluid and the velocity of sound. Correspondingly, a slope of a "plus characteristic" is everywhere equal to the sum of these velocities. Throughout this report we are dealing only with these two types of characteristic. In water the entropy change is considered to be negligible, therefore the third type, a streamline, which is a characteristic for the entropy, does not arise. Entropy changes are significant in region g, but not at the small distances from O' considered here. In the expansion region a fan of minus characteristics radiates from O'. This whole region is isentropic and can be described by two dependent variables u and a. Region g is bounded by a minus characteristic line on one side and a streamline on the other. This region is isentropic only to the extent to which its boundary with region e remains a minus characteristic. In the compressed water region the flow is entirely isentropic. In both regions three dependent variables u, p and a are used.

In the neighborhood of O', equations of motion are solved in series expansions to describe the state of motion. Further out in the field the method of characteristics is used. Wecken (1951) employs series expansions in the neighborhood of O' only up to terms or order $(t - t_0)^{\frac{1}{2}}$, where (t_0, r_0) are the coordinates of O', relying on numerical work to continue the field further out. This is inadequate to gain a full understanding of the initial field of disturbance, and, in particular, to prove conclusively that a second shock must always develop, the series must be taken to higher order terms. Berry and Holt (1954) made an investigation of the mathematical nature of the singularity in full. The flow is expanded in half-powers of the distance from O' in the t,r plane with coefficients depending on the angular coordinates. The expansions are not valid in the neighborhood of the minus characteristic through O' in region g. Near this line the expansions are modified by the extension of a technique due to Lighthill (1949). The method developed by Berry and Holt (1954) was generalized by Holt (1955) so that a spherical explosion in water may be considered. The result of their work is summarized in II.

In this report calculations are made to a spherical charge in water based on the method derived by Holt (1955). The series expansions are carried out separately in each of the regions and coefficients are computed according to the boundary conditions and equations of state in Holt (1956). One member each of the plus and minus characteristic lines are constructed.

The plus characteristic starts at a point on the detonation front very near O' and goes through the whole disturbed region. The minus characteristic goes radially from O' along the expansion-detonation boundary. With the positions and conditions of these two characteristics known, further out in the region other members of the characteristic families may be constructed in a step by step process. Numerical integration and an iterative method are employed and programmed on an IBM-CPC computer. It is found that even with the aid of the IBM computer the problem requires a considerable amount of time to solve completely. However, all results found are tabulated so that we are in a ready position to extend the work.

II. <u>Expansion in series near the origin of the main blast wave.</u>

The equations of momentum and continuity in unsteady spherical symmetric flow are, respectively

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0$$

$$\frac{\partial \rho}{\partial t} + \rho\frac{\partial u}{\partial r} + u\frac{\partial \rho}{\partial r} = -\frac{2\rho u}{r}$$

Throughout the analysis, all variables are made non-dimensional, The radial distance r and the time t are divided by their respective values at the surface of the sphere, the pressure p and the density ρ by their respective values at the detonation front and the fluid velocity u and velocity of sound a by the detonation velocity.

New variables ξ and θ are introduced and defined by

$$r = 1 - \xi^2 \sin\theta$$
$$t = 1 - \xi^2 \cos\theta$$

ξ^2 and θ are polar coordinates in the t,r plane based on O', with θ equal to the angle measured below O'M and ξ^2 the radial distance from O'.

The form of expansion to be assumed for each dependent variable is determined mainly by the known behavior in the detonation region. Using the results of Taylor (1950) and Döring (Döring and Burkhardt (1946)) the variables are analytic functions of ζ where $\zeta = \dfrac{\xi(\sin\theta - \cos\theta)^{\frac{1}{2}}}{(1 - \xi^2\cos\theta)^{\frac{1}{2}}}$.

Berry and Holt (1954) assume series expansions for u, a and p in the remaining regions, of the form

$$u = u_0(\theta) + u_1(\theta)\xi + u_2(\theta)\xi^2 + \dots$$
$$a = a_0(\theta) + a_1(\theta)\xi + a_2(\theta)\xi^2 + \dots$$
$$p = p_0(\theta) + p_1(\theta)\xi + p_2(\theta)\xi^2 + \dots$$

The expansions for each region are different, and are distinguished, one group from another, by using an appropriate suffix.

<u>The isentropic region</u>

In the isentropic region e, after introducing the auxiliary variable of state

$$\sigma = \frac{1}{k}\int \frac{dp}{\rho a}$$

where

$$k = \frac{D^2 \rho_d}{p_d}$$

the equations of momentum and continuity, when expressed in terms of the independent variables ξ and θ, may be written in the form

$$\begin{aligned}\xi\left\{\cos\theta + (u \pm a)\sin\theta\right\}\left(\frac{\partial u}{\partial \xi} \pm \frac{\partial \sigma}{\partial \xi}\right)& \\ - 2\left\{\sin\theta - (u \pm a)\cos\right\}\theta\left(\frac{\partial u}{\partial \theta} \pm \frac{\partial \sigma}{\partial \theta}\right) \mp \frac{4ua\xi^2}{1 - \xi^2\sin\theta} &= 0\end{aligned} \tag{2.1}$$

In equations (2.1) after substituting expansions of the dependent variables of the form

$$u_e = u_{oe}(\theta) + u_{1e}(\theta)\xi + u_{2e}(\theta)\xi^2 + \ldots$$

and equating corresponding coefficients of ξ, one group of equations are obtained for each order of coefficients. In this paper, the n'th order coefficient is referred to as the coefficient of ξ^n.

Terms independent of ξ in equations (2.1) give the relation between zero-order coefficients

$$u_{oe} + a_{oe} = \tan\theta \tag{2.2}$$

$$u_{oe} + \sigma_{oe} = \text{constant} \tag{2.3}$$

The terms of order ξ in (2.1) give the following relations between the first-order coefficients:

$$(u_{1e} - \sigma_{1e}) + 2(u_{1e} - a_{1e})(u'_{1e} - \sigma'_{1e})\cos^2\theta = 0 \tag{2.4}$$

$$\begin{aligned}\left\{\cos\theta + (u_{oe} + a_{oe})\sin\theta\right\}(u_{1e} + \sigma_{1e})& \\ - 2\left\{\sin\theta - (u_{oe} + a_{oe})\cos\theta\right\}(u'_{1e} + \sigma'_{1e}) &= 0\end{aligned} \tag{2.5}$$

where

$$u'_{oe} = \frac{du_{oe}}{d\theta}, \text{ etc.}$$

The relation between the coefficients of ξ^2 derived from (2.1) are

$$\begin{aligned} 2(u_{2e} - \sigma_{2e}) + 2(u'_{oe} - \sigma'_{oe})(u_{2e} - a_{2e})\cos^2\theta \\ + (u_{1e} - a_{1e})(u_{1e} - \sigma_{1e})\sin\theta\cos\theta \\ + 2(u_{1e} - a_{1e})(u'_{1e} - \sigma'_{1e})\cos^2\theta \\ + 4u_{oe}a_{oe}\cos\theta = 0 \end{aligned} \tag{2.6}$$

$$\begin{aligned} 2\left\{\cos\theta + (u_{oe} + a_{oe})\sin\theta\right\}(u_{2e} + \sigma_{2e}) \\ - 2\left\{\sin\theta - (u_{oe} + a_{oe})\cos\theta\right\}(u'_{2e} + \sigma'_{2e}) \\ + (u_{1e} + a_{1e})\left\{(u_{1e} + \sigma_{1e})\sin\theta + 2(u'_{1e} + \sigma'_{1e})\cos\theta\right\} \\ - 4u_{oe}a_{oe} = 0 \end{aligned} \tag{2.7}$$

Both variables σ_{oe} and a_{oe} are related to p_{oe} by the equation of state so that p_{oe} is defined implicitly by the equation

$$\sigma_{oe}(p_{oe}) + a_{oe}(p_{oe}) = \text{const.} - \tan\theta \tag{2.8}$$

The equation of state in region e may be written

$$p = p(\rho)$$

which becomes, on expansion

$$p_{oe} + p_{1e}\xi + p_{1e}\xi^2 + \ldots = p(\rho_{oe}) + ka_{oe}^2\rho_{1e}\xi$$

$$+ (ak_{oe}^2\rho_{2e} + \tfrac{1}{2}L_e\rho_{1e}^2)\xi^2 \ldots$$

where

$$ka_{oe}^2 = \left(\frac{dp}{d\rho}\right)_{oe}, \quad L_e = \left(\frac{d^2p}{d\rho^2}\right)_{oe}$$

We then have the relations

$$p_{1e} = ka_{oe}^2\rho_{1e}$$

$$p_{2e} = ka_{oe}^2\rho_{2e} + \tfrac{1}{2}L_e\rho_{1e}^2$$

Similarly, by expanding the equation

$$\frac{d\sigma}{dp} = \frac{1}{k\rho a}$$

we find that $p_{1e} = ka_{oe}\rho_{oe}\sigma_{1e}$

$$p_{2e} = k\left\{a_{oe}\rho_{oe}\sigma_{2e} + \tfrac{1}{2}\sigma_{1e}(a_{oe}\rho_{1e} + \rho_{oe}a_{1e})\right\}$$

From the equation $ka^2 = \frac{dp}{d\rho}$ we obtain

$$a_{1e} = \frac{L_e}{2ka_{1e}}\rho_{1e}$$

$$a_{2e} = \frac{1}{2ka_{oe}}\left\{L_e\rho_{2e} + \tfrac{1}{2}\rho_{1e}^2\left(\frac{d^3p}{d\rho^3}\right)_e - ka_{1e}^2\right\}$$

Combining the previous results, we find that

$$a_{1e} = \lambda_e\sigma_{1e}$$

where

$$\lambda = \frac{L\rho_o}{2ka_o^2}$$

The above relation is useful in solving u_{1e}, a_{1e} and σ_{1e} in terms of zero-order coefficients. A similar procedure can then be applied to equations (2.6) and (2.7) to find u_{2e}, a_{2e} σ_{2e}. The actual calculation of these coefficients will be presented in detail in III.

<u>The non-isentropic region</u>

Region c is, in general, entirely non-isentropic and region g is partially so. Much of the analysis of region g can be deduced as a simplification of that in region c.

After changing the independent variables r,t to ξ and θ, the equations of momentum and continuity are in the following form:

$$\xi\left\{\cos\theta + (u \pm a)\sin\theta\right\}\left(k\rho a\,\frac{\partial u}{\partial\xi} \pm \frac{\partial p}{\partial\xi}\right) - 2\left\{\sin\theta - (u \pm a)\cos\theta\right\}\left(k\rho a\,\frac{\partial u}{\partial\theta} \pm \frac{\partial p}{\partial\theta}\right) \mp \frac{4kua^2\rho\xi^2}{1-\xi^2\sin\theta} = 0 \qquad (2.8)$$

In the above equations we substitute expansions for each dependent variable of the type

$$u = u_{oc} + u_{1c}(\theta)\xi + u_{2c}(\theta)\xi^2 + \ldots$$

where u_{oc} is a constant. Equating coefficients of ξ and ξ^2, we then have a set of ordinary first-order differential equations for the coefficients of first and second order in turn. Solving these equations we obtain

$$u_{1c} = \frac{1}{k\rho_{oc}a_{oc}}(C_1\psi_1^{\frac{1}{2}} + C_2\psi_2^{\frac{1}{2}})$$

$$p_{1c} = C_1\psi_1^{\frac{1}{2}} - C_2\psi_2^{\frac{1}{2}}$$

$$\rho_{1c} = \frac{1}{ka^2}(C_1\psi_1^{\frac{1}{2}} - C_2\psi_2^{\frac{1}{2}})$$

$$a_{1c} = \frac{L_c}{2k^2a_{oc}^3}\left\{C_1\psi_1^{\frac{1}{2}} - C_2\psi_2^{\frac{1}{2}}\right\}$$

where

$$\psi_1 = \sin\theta - (u_{oc} + a_{oc})\cos\theta$$

$$\psi_2 = (u_{oc} - a_{oc}) - \sin\theta$$

$$L_c = \left(\frac{\partial^2 p}{\partial\rho^2}\right)_{oc}$$

and the C's are integration constants.

To find coefficients of ξ^2, auxiliary variables q and r are introduced and defined by

$$q = k\rho_o a_o u - p$$

$$r = k\rho_o a_o u + p$$

We obtain from the corresponding coefficients of ξ^2 in (2.8)

$$r_{2c} = \frac{\cos\theta}{k\rho_{oc}a_{oc}}\left\{2k^2u_{oc}a_{oc}^3\rho_{oc}^2 - (1+\lambda_c)(C_1^2 + C_2^2)\right\} + \frac{C_1C_2\psi_1}{3k\rho_{oc}a_{oc}^2}\left\{(1-\lambda_c)\left(\frac{\psi_2}{\psi_1}\right)^{3/2} - 3(1+\lambda_c)\left(\frac{\psi_2}{\psi_1}\right)^{\frac{1}{2}}\right\} - 2C_4\psi_1$$

$$q_{2c} = \frac{\cos\theta}{k\rho_{oc}a_{oc}}\left\{(1+\lambda_c)(C_1^2 + C_2^2) - 2ku_{oc}a_{oc}^3\rho_{oc}^2\right\} - \frac{C_1C_2\psi_2}{3k\rho_{oc}a_{oc}^2}\left\{(1-\lambda_c)\left(\frac{\psi_1}{\psi_2}\right)^{3/2} - 3(1+\lambda_c)\left(\frac{\psi_1}{\psi_2}\right)^{\frac{1}{2}}\right\} - 2C_5\psi_2$$

u_{2c} and p_{2c} may be solved from

$$q_{2c} = k\rho_o a_o u_{2c} - p_{2c}$$

$$r_{2c} = k\rho_o a_o u_{2c} - p_{2c}$$

The remaining second-order coefficients ρ_{2c} and a_{2c} can be deduced from the expansion of the equation of state $p = p(\rho, S)$ and $ka^2 = \partial p/\partial\rho$ at the initial condition. We obtain the relations

$$ka_1 = \frac{1}{2a_o} L\rho_1$$

$$ka_2 = \frac{1}{2a_o}(L\rho_2 + \tfrac{1}{2}P\rho_1^2 - ka_1^2)$$

where

$$P = \left(\frac{\partial^3 p}{\partial\rho^3}\right)_o$$

<u>The partially-isentropic region</u>

In region g, the situation is quite similar to that of region c. The zero-order coefficients are constants. The first-order coefficients are

$$u_{1g} = \frac{1}{k\rho_{og}a_{og}} (G_1\varphi_1^{\frac{1}{2}} - G_2\varphi_2^{\frac{1}{2}})$$

$$p_{1g} = G_1\varphi_1^{\frac{1}{2}} - G_2\varphi_2^{\frac{1}{2}}$$

$$\rho_{1g} = \frac{1}{ka_{og}^2} (G_1\varphi_1^{\frac{1}{2}} - G_2\varphi_2^{\frac{1}{2}})$$

$$a_{1g} = \frac{\lambda_g}{k\rho_{og}a_{og}} (G_1\varphi_1^{\frac{1}{2}} - G_2\varphi_2^{\frac{1}{2}})$$

The second-order coefficients are obtained through the auxiliary variables r and q.

$$r_{2g} = k\rho_{og}a_{og}u_{2g} + p_{2g}$$

$$= \frac{\cos\theta}{k\rho_{og}a_{og}}\left\{2k^2u_{og}a^3_{og}\rho^2_{og} - (1+\lambda_g)(G_1^2 + G_2^2)\right\}$$

$$+ \frac{G_1G_2\varphi_1}{3k\rho_{og}a^2_{og}}\left\{(1-\lambda_g)\left(\frac{\varphi_2}{\varphi_1}\right)^{3/2} - 3(1+\lambda_g)\left(\frac{\varphi_2}{\varphi_1}\right)^{\frac{1}{2}}\right\} - 2G_4\varphi_1$$

$$q_{2g} = k\rho_{og}a_{og}u_{2g} - p_{2g}$$

$$= \frac{\cos\theta}{k\rho_{og}a_{og}}\left\{(1+\lambda_g)(G_1^2 + G_2^2) - 2k^2u_{og}a^3_{og}\varphi^2_{og}\right\}$$

$$- \frac{G_1G_2\varphi_2}{3k\rho_{og}a^2_{og}}\left\{(1-\lambda_g)\left(\frac{\varphi_1}{\varphi_2}\right)^{3/2} - 3(1+\lambda_g)\left(\frac{\varphi_1}{\varphi_2}\right)^{\frac{1}{2}}\right\} - 2G_5\varphi_2$$

where

$$\varphi_1 = \sin\theta - (u_{og} + a_{og})\cos\theta$$

$$\varphi_2 = (u_{og} - a_{og})\cos\theta - \sin\theta$$

and the G's are integration constants. ρ_{2g} and a_{2g} can be solved in the same manner as ρ_{2g} and a_{2c}.

The expansion of r is valid throughout region g; but the expansion of q is not valid along the minus characteristic through O'. In order to overcome this difficulty, an independent variable z is introduced, where

$$\theta = \delta + z + \theta_{1g}(z)\xi + \theta_{2g}(z)\xi^2 + \dots$$

δ is the initial value of θ on the minus characteristic line through O'.

The new expression of q_{2g} in terms of z may be written

$$q_{2g}(z) = \frac{\cos(\delta+z)}{k\rho_{og}a_{og}}\left\{(1+\lambda_g)(G_1^2+G_2^2) - 2k^2u_{og}a_{og}^3\rho_{og}^2\right\}$$
$$- \frac{G_1G_2\varphi_2}{3k\rho_{og}a_{og}^2}\left\{(1-\lambda_g)\left(\frac{\varphi_1}{\varphi_2}\right)^{3/2} - 3(1+\lambda_2)\left(\frac{\varphi_1}{\varphi_2}\right)^{\frac{1}{2}}\right\}$$
$$- \frac{2}{3}\frac{G_1G_2}{k\rho_{og}a_{og}^2}\cos\delta(1-\lambda_g)(2a_{og})^{\frac{1}{2}}\sin^{-\frac{1}{2}}z\cos z - 2G_5\varphi_2$$

where φ_1 and φ_2 are evaluated for the argument $\delta + z$. At $z = 0$ or $\theta = \delta$

$$q_{2g} = \frac{\cos\delta}{k\rho_{og}a_{og}}\left\{(1+\lambda_g)(G_1^2+G_2)^2 - 2k^2u_{og}a_{og}^3\rho_{og}^2\right\} - 2G_5\varphi_2$$

III. Calculation of all coefficients in region e, g and c.

Region e

The equation of state in dimensionless form in region e is

$$\frac{1}{\rho} = 1.6309 - 1.2421p + 0.5165p + 0.09456p^{-5/6} \qquad (3.1)$$

From (3.1) we deduce the velocity of sound

$$a = \left(\frac{1}{k}\frac{dp}{d\rho}\right)^{\frac{1}{2}}$$
$$= \left(\frac{1}{k}\right)^{\frac{1}{2}}\frac{1.6309 - 1.2421p + 0.5165p^2 + 0.09465p^{-5/6}}{(1.2421 - 1.033p + 0.078875p^{-11/6})^{\frac{1}{2}}} \qquad (3.2)$$

and the auxiliary variable of state

$$= \frac{1}{k}\int_0^p \frac{dp}{a} = \left(\frac{1}{k}\right)^{\frac{1}{2}}\int_0^p (1.2421 - 1.033p + 0.078875p^{-11/6})^{\frac{1}{2}}dp$$

where

$$k = D^2\rho_d/p_d = 5.409$$

Knowing a and σ as functions of p we can next find θ as a function of the initial value of p in the expansion zone, p_{oe}, from the relation

$$\sigma_{oe}(p_{oe}) + a_{oe}(p_{oe}) = \text{constant} - \tan\theta \qquad (3.3)$$

The constant is immediately found from data at the detonation front at which $p_{oe} = 0.6132$, $\theta = 2.8876$, $\sigma_{oe}(p_{oe}) = 1.49186$, $a_{oe} = 0.5787$, constant $= 1.81101$.

All the initial values of dependent variables in the expansion zone are then defined implicitly as functions of θ.

To obtain the first-order coefficients, consider first equation (2.5), the solution of which gives us the ratio of $u_{1e}(\theta) + \sigma_{1e}(\theta)$ to $u_{1e}(\alpha) + \sigma_{1e}(\alpha)$. We may write

$$u_{1e}(\theta)+\sigma_{1e}(\theta)=\left\{u_{1e}(\alpha)+\sigma_{1e}(\alpha)\right\}e^{\frac{1}{2}\int^{\theta}\frac{\left\{\cos\theta + (u_{oe}+a_{oe})\sin\theta\right\}}{\left\{\sin\theta - (u_{oe}+a_{oe})\cos\theta\right\}}d\theta} \qquad (3.4)$$

α is the initial value of the boundary between e and d.

In order to determine $u_{1e}(\alpha) + \sigma_{1e}(\alpha)$, we take the result of Döring that in the detonation region u and σ are functions of ζ and may be written

$$u = u_o + u_1\zeta + u_2\zeta^2 + \ldots$$
$$\sigma = \sigma_o + \sigma_1\zeta + \sigma_2\zeta^2 + \ldots$$

where the coefficients are constant and

$$\zeta^2 = \frac{\xi^2(\sin\theta - \cos\theta)}{1 - \xi^2\cos\theta}$$

At the boundary between regions e and g

$$\theta = \alpha + \beta_1\xi + \beta_2\xi^2 + \dots$$

Now, expand ζ in Taylor's series about the point

$$\theta = \alpha,$$

we obtain

$$\zeta = \xi(\sin\alpha - \cos\alpha)^{\frac{1}{2}} + \xi^2 \frac{\beta_1(\cos\alpha + \sin\alpha)}{2(\sin\alpha - \cos\alpha)^{\frac{1}{2}}} + \dots$$

Substitute into $u + \sigma$ to get

$$u + \sigma = (u_o + \sigma_o) + (u_1 + \sigma_1)(\sin\alpha - \cos\alpha)^{\frac{1}{2}}\xi$$
$$+ [(u_2 + \sigma_2)(\sin\alpha - \cos\alpha) + (u_1 + \sigma_1)\frac{\beta_1(\cos\alpha + \sin\alpha)}{2(\sin\alpha - \cos\alpha)^{\frac{1}{2}}}]\xi^2 + \dots$$

But in region e we have

$$u_e = u_{oe}(\theta) + u_{1e}(\theta)\xi + u_{2e}(\theta)\xi^2 + \dots$$
$$\sigma_e = \sigma_{oe}(\theta) + \sigma_{1e}(\theta)\xi + \sigma_{2e}(\theta)\xi^2 + \dots$$

Expand $u_e + \sigma_e$ about $\theta - \alpha$ and compare coefficients of same powers of ξ with those in $u + \sigma$. We obtain

$$u_o + \sigma_o = u_{oe}(\alpha) + \sigma_{oe}(\alpha)$$

$$(u_1 + \sigma_1)(\sin\alpha - \cos\alpha)^{\frac{1}{2}} = u_{1e}(\alpha) + \sigma_{1e}(\alpha)$$

Considering equation (2.4) and taking into account the previous results we obtain

$$u_{1e}(\theta) + a_{1e}(\theta) = \sigma_{1e}(\theta)\,\frac{6(1+\lambda)}{5+\lambda}$$

We can now find all first-order coefficients u, σ and a as functions of θ.

To obtain second-order coefficients we first solve the three simultaneous equations for u_2, a_2 and σ_2

$$u_2 - \sigma_2 = \frac{-2u_o a_o}{u_o - a_o - 1} \qquad (3.5)$$

$$(u_2 + a_2 + 1) + (1+\lambda)(u_2 + \sigma_2) = 2(a_o + \lambda u_o) \qquad (3.6)$$

$$a_2 = \lambda\sigma_2 + \tfrac{1}{2}\mu\sigma_1^2 \qquad (3.7)$$

where $\mu = \dfrac{d\lambda}{d\sigma}$

The results are

$$u_2 = \frac{1}{3(1+\lambda)}\left\{a_o - \tfrac{1}{2}\mu\sigma_1^2 + 4\lambda u_o\right\}$$

$$a_2 = \frac{\lambda}{3(1+\lambda)}\left\{a_o - 3u_o - \tfrac{1}{2}\mu\sigma_1^2 + \lambda u_o\right\} + \tfrac{1}{2}\mu\sigma_1^2$$

$$\sigma_2 = \frac{1}{3(1+\lambda)}\left\{a_o - 3u_o - \tfrac{1}{2}\mu\sigma_1^2 + \lambda u_o\right\}$$

Solving equation (2.7) gives us

$$[u_{2e}(\theta) + \sigma_{2e}(\theta)] - [u_{2e}(\alpha) + \sigma_{2e}(\alpha)] = e^{-\int_\alpha^\theta g(\theta)d\theta}\int_\alpha^\theta h(\theta)e^{\int_\alpha^\theta g(\theta)d\theta}\,d\theta$$

where

$$g(\theta) = \frac{\{\cos\theta + (u_{oe} + a_{oe})\sin\theta\}}{\{\sin\theta - (u_{oe} + a_{oe})\cos\theta\}}$$

$$h(\theta) = \frac{(u_{1e}+a_{1e})\left\{(u_{1e}+\sigma_{1e})\sin\theta + 2(u_{1e}'+\sigma_{1e}')\cos\theta\right\} - 4u_{oe}a_{oe}}{\sin\theta - (u_{oe}+a_{oe})\cos\theta}$$

Solving equation (2.6) gives us

$$u_{2e} - \sigma_{2e} = \tfrac{1}{2}\left\{\frac{u_{1e}-\sigma_{1e}}{u_{1e}-a_{1e}}(u_{2e}-a_{2e}) - (u_{1e}-a_{1e})\cos\theta\,[(u_{1e}-\sigma_{1e})\sin\theta + 2(u_{1e}'-\sigma_{1e}')\cos\theta] - 4u_{oe}a_{oe}\cos\theta\right\} \qquad (3.9)$$

With the equation

$$u_{2e} + \sigma_{2e} = -\left(\frac{1+\lambda}{1-\lambda}\right)(u_{2e} - \sigma_{2e}) + \frac{2}{1-\lambda}(u_{2e} - a_{2e}) + \frac{\mu\sigma_{1e}^2}{1-\lambda} \qquad (3.10)$$

we then have three simultaneous equations to solve u_{2e}, $_{2e}$ and a_{2e} explicitly as functions of θ.

Region c

The equation of state in region c is

$$p = \frac{B}{p_d}\left\{\left(\frac{\rho}{\rho_o}\right)^n - 1\right\}$$

where n, B and ρ_o are constants determined by Richardson, Arons and Halverson (1947)

$$n = 7$$

$$B = 3.311 \times 10^9 \text{ dyn/cm}^2$$

$$\rho_o = 1.007 \text{ g/cm}^3$$

$$p_d = 24.746 \times 10^{10} \text{ dyn/cm}^2.$$

Equation (3.11) provides us with the corresponding values of L_c, λ_c etc.

The computation involved in solving u_c and a_c is much simpler than those in region e. The required integration constants and boundary conditions are supplied by Holt (1956) and coefficients of first and second-order are evaluated according to the expressions in II.

Region g.

Coefficients in region g are evaluated in a similar manner to those in region c using the expressions listed in II. Since this is a uniform region as far as certain properties are concerned, only first and second-order coefficients are functions of θ and values of λ_g, L_g and P_g are constants equal to the values of λ_e, L_e and P_e at the boundary.

All values of u_e, a_e, σ_e, u_c, u_g and a_g are tabulated as functions of θ in V. All necessary data for the computation obtained from Holt (1956) are given in the appendix.

IV. The construction of characteristic lines.

The minus characteristic line.

A fan of minus characteristic lines goes through O' in the expansion region, among them is the boundary between regions e and d. The boundary is expressed by the equation

$$\theta = 2.5997 + 0.6197\xi - 1.8084\xi^2 + \ldots$$

Our problem now is to determine the fluid properties along this line.

We know, from the result of Döring, that in the detonation region, the dependent variables are expressed in the form

$$u = u_o + u_1\zeta + u_2\zeta^2 + \dots$$

Substitute them into the equations

$$(u \pm a - z)\,\frac{d(u \pm \sigma)}{dz} = \mp\,\frac{2ua}{z} \qquad (4.1)$$

where $z = 1 - \zeta^2$.

We can determine a point of known properties by choosing a small value of ξ, thus of θ and ζ. From this point on we can determine all the properties along the bounding characteristic by integrating (4.1) numerically.

The initial point is chosen at $\zeta = 0.1$ or $z = 0.99$, u, a and σ are evaluated respectively.

Equations (4.1) are replaced by simple differential equations (4.2) as a first approximation

$$\frac{(u_2 + \sigma_2) - (u_1 + \sigma_1)}{z_2 - z_1} = -\,\frac{2u_1a_1}{z_1}\,\frac{1}{u_1 + a_1 - z_1}$$

$$\frac{(u_2 - \sigma_2) - (u_1 - \sigma_1)}{z_2 - z_1} = \frac{2u_1a_1}{z_1}\,\frac{1}{u_1 - a_1 - z_1} \qquad (4.2)$$

z_2 is a value chosen such that rapid convergence ensued.

u_2 and σ_2 can be determined from (4.2) and a_2 may be obtained from the relation

$$\sigma = \frac{1}{k}\int_0^p \frac{dp}{\rho a}$$

A better approximation of u_2 and σ_2 can be achieved by writing the difference equation in the form

$$\frac{(u_2+\sigma_2)-(u_1+\sigma_1)}{z_2-z_1} = -\frac{1}{2}\left\{\frac{2u_1a_1}{z_1(u_1+a_1-z_1)} + \frac{2u_2a_2}{z_2(u_2+a_2-z_2)}\right\}$$
$$\frac{(u_2-\sigma_2)-(u_1-\sigma_1)}{z_2-z_1} = \frac{1}{2}\left\{\frac{2u_1a_1}{z_1(u_1-a_1-z_1)} + \frac{2u_2a_2}{z_2(u_2+a_2-z_2)}\right\} \tag{4.3}$$

Values of u_2, a_2 from the first approximation are used, and the new value a_2 may be found by using the new value of σ_2. This process may be carried out a few more times to obtain enough accuracy. The values of u, a and σ at different z or ζ are tabulated in Table 4, V.

The plus characteristic line

The "plus" characteristic family lie across the flow field from the boundary between e and d to the main shock. One member of the family lying very near O' is constructed in the following manner.

The governing equation

$$\frac{d\xi}{d\theta} = \frac{\frac{1}{2}\xi\{1 + (u + a)\tan\theta\}}{u + a - \tan\theta} \tag{4.4}$$

is replaced by a simple difference equation. A small value of ξ on the boundary between e and g is chosen to be used as an initial point. Having found all the coefficients of series expansion in regions e, g and c, we are able to integrate the above equation numerically in the same manner as the minus characteristics. This process gives the position and condition of this particular "plus" characteristic. These data are given in Table 5, V.

The construction of further characteristic lines

Having determined one "plus" and one "minus" characteristic line, we are in a position to construct the whole flow field by the method of characteristics.

Due to limit of time the whole work is not tried at the present stage. However, a plus characteristic line has been computed in region e by IBM CPC as an illustration of the method.

Using a modified method of Berry, Butler and Holt (1955) we introduce new independent variables x and y by the relations

$$x = t - 1$$
$$y = r - 1$$

The characteristic lines are defined by

$$\frac{dy}{dx} = u \pm a \qquad (4.5)$$

The characteristic equations are then

$$du \pm d\sigma = \mp \frac{2au}{1 + y} dx \qquad (4.6)$$

In numerical work equations (4.5) and (4.6) are replaced by simple difference equations and an iterative method may be employed as in the construction of the "minus" and "plus" characteristics.

Figure 2 shows a characteristic quadrangle in which 12 and 13 are given segments of known "minus" and "plus" characteristics respectively and the point 4 is to be constructed. A first approximation is made by calculating the position of point 4 as the intersection of tangents to characteristics at points 2 and 3, using equation (4.5), then finding values of u and σ at 4 by solving equations (4.6) as simple difference equations. Better approximations may be obtained by iteration in the same manner as described above. The whole process is programmed on an IBM CPC computer in a continuous manner. A point lying out on the minus characteristic was picked to be point 2, all the needed known properties associated with this point are stored in the machine, likewise are the data of a chosen point 3 on the known plus characteristics. The two difference equations (4.5) are solved to give the values of x, y and then u and σ in (4.6). The value of σ is then used to calculate its corresponding value of p from the relation

$$\frac{d\sigma}{dp} = \frac{1}{k\rho a} \qquad (4.7)$$

Equation (4.7) is replaced by a simple difference equation using either point 2 or 3 as one end point. The value of p is iterated according to (4.7) and its final value is used to calculate a. After the positions and properties of point 4 are found, the machine automatically stores everything connecting 4 into those previously containing 2 and a set of data of a new point 3 is fed in and the process is repeated. A line containing roughly 25 points may be constructed in a matter of six hours. It is thus felt that CPC is not the most suitable equipment for doing this type of work, both time and costwise. The value of the "plus" characteristic is given in Table 6, V.

V. Tables

Table 1. Coefficients of series expansion in region e

u

θ	u_{0e}	u_{1e}	u_{2e}	θ	u_{0e}	u_{1e}	u_{2e}
2.5997	.1990	-.3671	-.028467	2.81	.2705	-.2431	.114960
2.60	.1990	-.3671	-.028397	2.82	.2759	-.2347	.129732
2.61	.2012	-.3635	-.026864	2.83	.2817	-.2272	.144403
2.62	.2032	-.3599	-.024931	2.84	.2875	-.2188	.161040
2.63	.2054	-.3559	-.022719	2.85	.2941	-.2106	.178349
2.64	.2076	-.3518	-.019769	2.86	.3005	-.2028	.195608
2.65	.2101	-.3475	-.016512	2.87	.3067	-.1945	.214380
2.66	.2127	-.3428	-.012531	2.88	.3138	-.1873	.231502
2.67	.2153	-.3382	-.008114	2.8876	.31915	-.1819	.250759
2.68	.2181	-.3329	-.003510				
2.69	.2211	-.3275	+.001988				
2.70	.2242	-.3220	+.007069				
2.71	.2275	-.3159	+.014615				
2.72	.2309	-.3100	+.020684				
2.73	.2345	-.3033	+.028998				
2.74	.2384	-.2966	.037045				
2.75	.2424	-.2903	.045066				
2.76	.2465	-.2834	.054294				
2.77	.2508	-.2763	.064760				
2.78	.2555	-.2691	.075793				
2.79	.2602	-.2611	.087285				
2.80	.2654	-.2516	.100888				

Table I (continued)

a^{θ}

θ	a_{0e}	a_{1e}	a_{2e}	θ	a_{0e}	a_{1e}	a_{2e}
2.5997	.8010	-.7893	.600824	2.81	.6152	-.2488	.36240
2.60	.8010	-.7893	.597736	2.82	.6096	-.2323	.37880
2.61	.7890	-.7481	.55900	2.83	.6045	-.2186	.38830
2.62	.7778	-.7103	.51145	2.84	.5995	-.2033	.40273
2.63	.7666	-.6750	.47830	2.85	.5947	-.1894	.41910
2.64	.7560	-.6414	.44500	2.86	.5900	-.1767	.43800
2.65	.7458	-.6107	.42480	2.87	.5850	-.1635	.45850
2.66	.7355	-.5793	.40460	2.88	.5810	-.1522	.47892
2.67	.7255	-.5512	.38798	2.876	.5787	-.1440	.49168
2.68	.7159	-.5226	.37364				
2.69	.7065	-.4954	.36050				
2.70	.6970	-.4709	.34950				
2.71	.6886	-.4459	.34150				
2.72	.6800	-.4234	.33450				
2.73	.6713	-.4004	.32800				
2.74	.6632	-.3790	.32450				
2.75	.6555	-.3605	.31950				
2.76	.6480	-.3414	.31950				
2.77	.6410	-.3229	.32400				
2.78	.6340	-.3051	.33150				
2.79	.6275	-.2866	.34200				
2.80	.6210	-.2659	.35150				

Table I (continued)

σ_e

θ	σ_{0e}	σ_{1e}	σ_{2e}	θ	σ_{0e}	σ_{1e}	σ_{2e}
2.5997	1.612	-.1294	-.24908	2.81	1.5400	-.2325	-.41922
2.60	1.6118	-.1294	-.24916	2.82	1.5345	-.2395	-.43587
2.61	1.6096	-.1324	-.25152	2.83	1.5287	-.2456	-.45249
2.62	1.6075	-.1353	-.25431	2.84	1.5230	-.2526	-.47115
2.63	1.6052	-.1386	-.25744	2.85	1.5170	-.2594	-.49054
2.64	1.6030	-.1419	-.26133	2.86	1.5104	-.2657	-.50997
2.65	1.6008	-.1454	-.26556	2.87	1.5038	-.2725	-.53099
2.66	1.5982	-.1493	-.27057	2.88	1.4970	-.2782	-.55042
2.67	1.5952	-.1531	-.27606	2.8876	1.4918	-.2824	-.57148
2.68	1.5925	-.1574	-.28177				
2.69	1.5896	-.1619	-.28842				
2.70	1.5865	-.1664	-.29470				
2.71	1.5830	-.1715	-.30350				
2.72	1.5800	-.1764	-.31087				
2.73	1.5764	-.1820	-.32052				
2.74	1.5727	-.1876	-.32996				
2.75	1.5690	-.1928	-.33942				
2.76	1.5650	-.1985	-.35015				
2.77	1.5605	-.2044	-.36218				
2.78	1.5560	-.2104	-.37482				
2.79	1.5510	-.2171	-.38799				
2.80	1.5456	-.2253	-.40333				

Table 2. Coefficients of series expansion in region g

θ	u_{og}	u_{1g}	u_{2g}	θ	u_{og}	u_{1g}	u_{2g}
			u				
2.8876	.3191	-.1881348	.0321892	3.24	.3191	-.140979	.167320
2.8880	.3191	-.18753696	-.0014837	3.26	.3191	-.138320	.174408
2.8885	.3191	-.18720721	.0099878	3.28	.3191	-.135612	.181429
2.8890	.3191	-.18695451	.0146487	3.30	.3191	-.132855	.188381
2.890	.3191	-.18654632	.0193006	3.32	.3191	-.130044	.195259
2.900	.3191	-.184051	.0309208	3.34	.3191	-.127178	.202060
2.920	.3191	-.180703	.0410614	3.36	.3191	-.124252	.208783
2.940	.3191	-.177898	.0496793	3.38	.3191	-.121263	.215423
2.960	.3191	-.175308	.0579384	3.40	.3191	-.118209	.221978
2.980	.3191	-.172829	.0660172	3.42	.3191	-.115083	.228445
3.000	.3191	-.170410	.0739900	3.44	.3191	-.111884	.234819
3.02	.3191	-.168025	.0818851	3.4405	.3191	-.110173	.238129
3.04	.3191	-.165653	.0897163				
3.06	.3191	-.163283	.0974877				
3.08	.3191	-.160906	.10865214				
3.10	.3191	-.158514	.11613660				
3.12	.3191	-.156103	.12358839				
3.14	.3191	-.153666	.13100068				
3.16	.3191	-.151200	.13836904				
3.18	.3191	-.148702	.14568949				
3.20	.3191	-.146167	.15295665				
3.22	.3191	-.143594	.16016830				

Table 2 (continued)

a

θ	a_{0g}	a_{1g}	a_{2g}	θ	a_{0g}	a_{1g}	a_{2g}
2.8876	.5787	-.0959742	-.24267430	3.24	.5787	-.089799	-.330294
2.8880	.5787	-.09625375	-.21407971	3.26	.5787	-.088921	-.333795
2.8885	.5787	-.09639008	-.22933902	3.28	.5787	-.088003	-.337153
2.8890	.5787	-.09647710	-.23234037	3.30	.5787	-.087045	-.340371
2.890	.5787	-.09663147	-.23540690	3.32	.5787	-.086046	-.343445
2.900	.5787	-.097259	-.24372671	3.34	.5787	-.085005	-.346374
2.920	.5787	-.097643	-.25143223	3.36	.5787	-.083921	-.349157
2.94	.5787	-.097701	-.25783400	3.38	.5787	-.082793	-.351792
2.96	.5787	-.097602	-.26380211	3.40	.5787	-.081620	-.354281
2.98	.5787	-.097398	-.26949212	3.42	.5787	-.080400	-.356619
3.00	.5787	-.097113	-.27497359	3.44	.5787	-.079132	-.358808
3.02	.5787	-.096763	-.28027525	3.4505	.5787	-.078446	-.359897
3.04	.5787	-.096354	-.28541352				
3.06	.5787	-.095894	-.29039747				
3.08	.5787	-.095385	-.29736800				
3.10	.5787	-.094831	-.30194900				
3.12	.5787	-.09235	-.30640000				
3.14	.5787	-.093596	-.31071900				
3.16	.5787	-.092916	-.31490500				
3.18	.5787	-.092196	-.31895800				
3.20	.5787	-.091437	-.32287500				
3.22	.5787	-.090638	-.32665300				

Table 3. Coefficients of series expansion in region c

u				a			
θ	u_{0c}	u_{1c}	u_{2c}	θ	a_{0c}	a_{1c}	a_{2c}
3.4505	.3191	-.108258	-.002841	3.4505	.9971	-.004631	-.012636
3.46	.3191	-.107620	-.006328	3.46	.9971	-.004595	-.012609
3.48	.3191	-.106246	-.013667	3.48	.9971	-.004516	-.012546
3.50	.3191	-.104829	-.021001	3.50	.9971	-.004435	-.012479
3.52	.3191	-.103365	-.028321	3.52	.9971	-.004352	-.012407
3.54	.3191	-.101855	-.035641	3.54	.9971	-.004267	-.012329
3.56	.3191	-.100297	-.042940	3.56	.9971	-.004180	-.012247
3.58	.3191	-.098688	-.050223	3.58	.9971	-.004090	-.012159
3.60	.3191	-.097026	-.057485	3.60	.9971	-.003998	-.012066
3.62	.3191	-.095310	-.064724	3.62	.9971	-.003903	-.011968
3.64	.3191	-.093537	-.071936	3.64	.9971	-.003806	-.011865
3.66	.3191	-.091703	-.079120	3.66	.9971	-.003705	-.011758
3.68	.3191	-.089805	-.086272	3.68	.9971	-.003602	-.011645
3.70	.3191	-.087841	-.093389	3.70	.9971	-.003495	-.011527
3.72	.3191	-.085804	-.100468	3.72	.9971	-.003385	-.011405
3.74	.3191	-.083692	-.107506	3.74	.9971	-.003271	-.011277
3.76	.3191	-.081495	-.114500	3.76	.9971	-.003153	-.011145
3.78	.3191	-.079210	-.121448	3.78	.9971	-.003030	-.011008
3.7881	.3191	-.078258	-.124248	3.7881	.9971	-.002979	-.010951

Table 4. Second plus characteristic in region e

x	y	u	a	σ
.00491874	-.00272657	.180797	.714320	1.592003
.0049530	-.00269038	.183178	.706212	1.589612
.00499898	-.00265519	.185512	.698782	1.587268
.00503931	-.00261963	.188004	.691360	1.584765
.00508140	-.00258272	.190763	.683703	1.581995
.00512419	-.00254540	.193719	.676083	1.579028
.00516652	-.00250866	.196649	.669068	1.576087
.00520991	-.00247119	.199786	.662088	1.572938
.00525443	-.00243389	.203076	.655302	1.569637
.00529901	-.00239470	.206693	.648406	1.566008
.00534324	-.00235694	.210101	.642392	1.562587
.00538979	-.00231731	.213942	.636122	1.558734
.00543675	-.00227743	.217985	.630043	1.554679
.00548351	-.00223782	.222063	.624394	1.550587
.00553095	-.00219769	.226366	.618907	1.546271
.00557925	-.00215688	.231030	.613448	1.541593
.00562902	-.00211486	.235900	.608230	1.536708
.00567888	-.00207277	.241053	.603189	1.531542

Table 4 (continued)

x	y	u	a	σ
.00573133	-.00202848	.246718	.59148	1.525861
.00578250	-.00198521	.252399	.593557	1.520165
.00583420	-.00194143	.25817~~7~~6	.589306	1.514371
.00588693	-.00189669	.264273	.585222	1.508258
.00593944	-.00185203	.270360	.581503	1.502155
.00599500	-.00180462	.276995	.577802	1.495503
.00605073	-.00175688	.283813	.574329	1.488666
.00613273	-.00168628	.294272	.569529	1.478180
.00618400	-.00164190	.300613	.566875	1.471822

Table 5. Position of an arbitrary plus characteristic near O'

θ	ξ	θ	ξ	θ	ξ	θ	ξ
2.62	.036675	2.85	.0387349	3.24	0.0466291	3.68	0.0698348
2.63	.0367317	2.86	.0388466	3.26	0.0472913	3.70	0.0716605
2.64	.0367915	2.87	.0389712	3.28	0.0479925	3.72	0.0736531
2.65	.0368551	2.88	.0390987	3.30	0.0487359	3.74	0.0758384
2.66	.0369207	2.8848	.03916207	3.32	0.0495252	3.76	0.0782476
2.67	.0369892	2.89	.0392295	3.34	0.0503645	3.78	0.0809201
2.68	.0370607	2.90	.0393636	3.36	0.0512586		
2.69	.0371350	2.92	.0396421	3.38	0.0522130		
2.70	.0372124	2.94	.0399347	3.40	0.0532337		
2.71	.0372937	2.96	.0402420	3.42	0.0543281		
2.72	.0373724	2.98	.0405647	3.44	0.0555045		
2.73	.0374574	3.00	.0409037	3.446	0.0558721		
2.74	.0375465	3.02	.0412597	3.46	0.05660		
2.75	.0376385	3.04	.0416336	3.48	0.0574515		
2.76	.0377334	3.06	.0420264	3.50	0.0583548		
2.77	.0378313	3.08	0.0424392	3.52	0.0593143		
2.78	.0379320	3.10	0.0428732	3.54	0.0603354		
2.79	.0380356	3.12	0.0433296	3.56	0.0614238		
2.80	.0381421	3.14	0.0438098	3.58	0.0625864		
2.81	.0382422	3.16	0.0443155	3.60	0.0638311		
2.82	.0383535	3.18	0.0448482	3.62	0.0651667		
2.83	.0384683	3.20	0.0454100	3.64	0.0666039		
2.84	.0385359	3.22	0.0460028	3.66	0.0681551		

Table 6. Minus characteristic

x	y	u	a	σ
.00116620	-.00067019	.1899	.7524	1.6025
.00492125	-.00267994	.1800	.7162	1.5928
.00969330	-.00525267	.1725	.6888	1.5832
.0172790	-.00925181	.1639	.6610	1.5712
.0270417	-.0143749	.1542	.6400	1.5590
.104652	-.063729	.1045	.5810	1.4970
.193669	-.158140	.0527	.5534	1.4300

Appendix

(From Holt (1956))

Table 1. Initial values of dependent variables

	air	water
$p_{0c} = p_{0g} = p_{0e}(\delta)$	2.763×10^{-3}	6.132×10^{-1}
$u_{0c} = u_{0g} = u_{0e}(\delta)$	9.219×10^{-1}	3.191×10^{-1}
$a_{0g} = a_{0e}(\delta)$	9.989×10^{-2}	5.787×10^{-1}
a_{0c}	2.957×10^{-1}	9.971×10^{-1}

Table 2. Constants of integration

	air	water		air	water
C_1	-1.738×10^{-3}	-5.504×10^{-1}	C_4	-3.342×10^{-3}	-9.457×10^{-1}
C_2	-3.797×10^{-4}	-5.291×10^{-2}	C_5	9.428×10^{-3}	9.619×10^{-1}
C_3	-6.433×10^{-1}	0	C_6	6.663	θ
G_1	-7.243×10^{-3}	-5.604×10^{-1}	G_4	1.274×10^{-2}	-9.056×10^{-2}
G_2	-9.580×10^{-3}	9.264×10^{-2}	G	5.444×10^{-2}	-1.271

Table 3. Equations of boundaries

	Θ in air	Θ in water
characteristic OA	$2.5997+0.6197\xi-1.8034\xi^2\ldots$	$2.5997+0.6197\xi-1.804\xi^2\ldots$
second shock	[illegible]296$-0.02304\xi+0.001878\xi^2\ldots$	$2.8876-0.06984\xi+0.01290\xi^2\ldots$
separation surface	$2.8364-0.04335\xi-0.03646\xi^2\ldots$	$3.4505-0.08088\xi-0.006950\xi^2\ldots$
main shock	$3.9348-0.02744\xi-0.05347\xi^2\ldots$	$3.7831-0.06595\xi-0.08373\xi^2\ldots$

References

Doring, W., and Burkhardt, G., 1946 Wright Field Tech. Rep. F-TS-a227-1A (Brown University Translation).

Richardson, J. M., Arons, A. B., and Halverson, R. R., 1947 J. Chem. Phys. 15, 785-794.

Lighthill, M. J., 1949 Phil Mag. (7) 40, 311, 1179-1201.

Taylor, Sir Geoffrey, 1950 Proc. Roy. Soc. A, 200, 235-247

Shitham, G. B., 1950 Proc. Roy. Soc. A, 203, 571-581.

Wocken, F., 1951 Mem. 14m Lab. Recherches Tech. Saint Louis.

Berry, F. J. and Holt, M., 1954 Proc. Roy. Soc. A, 224, 236-251.

Berry, F. J., Butler, D. S., and Holt, M., 1954 Proc. Roy. Soc. A, 227, 258-270.

Holt, M., 1956 Proc. Roy. Soc. A, 234, 110-115.

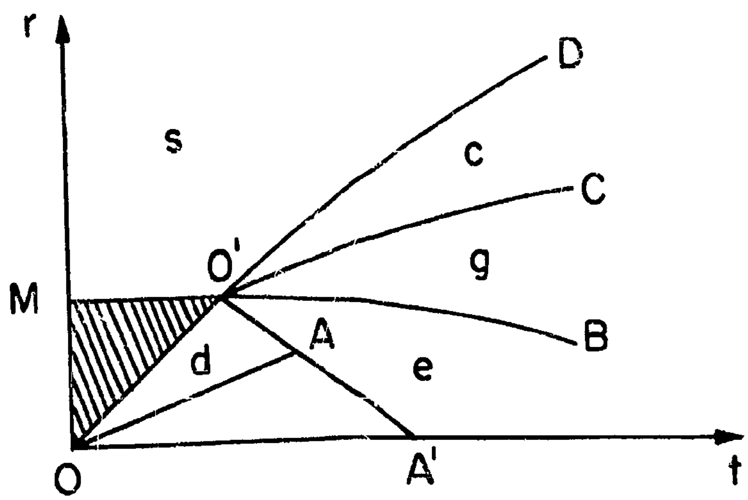
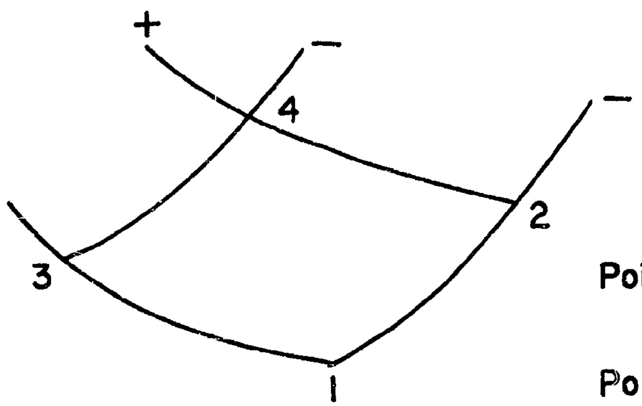


FIG. I



Points 2,4 are on the same "+" line

Points 3,4 are on the same "−" line

FIG. 2